Estimation of Asset Value Changes Using Stochastic Models for Capital Market Investments in Oando Plc, Nigeria

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Abstract
This paper simultaneously applied Stochastic Delay Differential Equations (SDDEs) alongside Stochastic Differential Equations (SDEs) through both multiplicative return structures within an emerging capital market context such as Nigeria. However, no prior study including Amadi & Okpoye (2022) and Okpoye et al. (2023) had simultaneously incorporated time-dependent delay parameters into multiplicative and structures, nor applied such a framework within an emerging capital market context like Nigeria. It was observed that investors often engage in delayed asset liquidation, expecting appreciation over time; however, the quantitative implications of such delay under stochastic market uncertainty remained insufficiently understood. Simulations were performed for varying levels of delay parameters and return rates (1.0000 and 2.0000), under constant volatility, to determine whether delayed strategies generate statistically superior financial outcomes in comparison to immediate asset liquidation. Results indicated that the presence of delay significantly enhances asset values under both return structures, with delayed investments yielding consistently higher returns than non-delayed ones. The analysis further revealed that an increase in return rates amplifies these gains, confirming that delay and rate of return interaction contribute positively to wealth accumulation. Goodness-of-fit tests using the Kolmogorov–Smirnov (KS) procedure showed that asset values with delay and without delay do not originate from the same distribution at a 1% significance level. This statistical divergence suggests that delay is not only financially advantageous but also structurally alters the stochastic behaviour of asset values, implying higher risk–return sensitivity. Moreover, graphical results corroborated these findings by displaying upward trajectories for delayed assets, contrasted with low or stagnant returns for non-delayed strategies. Overall, the study contributed to financial modelling literature by demonstrating how delay-driven investment strategies can serve as profit-maximizing approaches in uncertain market environments.

Keywords: *Stochatic systems, Stock Prices, KS, Delay, Multiplicative and Investments*

1.1 Introduction

The estimation of asset value changes in the capital market is a crucial aspect of financial modeling, enabling investors, analysts, and policymakers to assess risk, return and make informed decisions. Given the inherent uncertainty and randomness in financial markets, stochastic models have become widely accepted tools for capturing the dynamic behavior of asset prices over time. These models employ random processes to account for unpredictable market fluctuations, allowing for more realistic simulations of future asset values (Hull, 2022). The application of stochastic models, such as Geometric Brownian Motion (GBM), Jump Diffusion models, and stochastic volatility models, offers valuable insights into price dynamics by incorporating factors like volatility, drift, and random shocks (Shreve, 2014). For instance, GBM assumes that asset prices follow a continuous path driven by both deterministic trends and random noise, making it an effective model for capturing the uncertainty in stock prices. However, real-world markets often exhibit jumps and sudden shifts due to unforeseen events, which necessitate the use of Jump Diffusion models (Merton, 2017). Additionally, in sectors like energy, where companies such as Oando PLC operate, commodity prices often follow mean-reverting patterns, requiring specialized stochastic models that account for such behaviors (Schwartz, 2019). A lot of work has been done on modeling of inventory market expenses are as follows, Loko et al. (2023) carried out research on the effect of Fourier series expansion on the solution of Stochastic Differential Equation (SDE). They obtained detailed measures that govern the price function of return rate for capital investments periodically. In providing solution to the proposed model, price functions were adopted as drift (return rate) parameter which according to their propositions follows various patterns in obtaining solutions. They concluded by showing the effects of the significant parameters demonstrating graphically for purposes of decision making. Okpoye et al. (2023) worked on an empirical assessment of asset value function for capital market price changes. Amadi and Wobo (2022) carried a study on: A Mathematical Model Analysis for Estimating Stock Market Price Changes. In the study, the investigators examined various techniques for estimating the parameters of Weibull distribution, they used the Mean Square Error (MSE) technique as a criterion for choosing the best model. Amadi and Okpoye (2022) Researched on: Application of stochastic model in estimation of stock return rates in capital market investments. In a similar research work by Amadi and Vivian (2022) on: A stochastic analysis of stock price variation assessments in Oando Nigeria Plc, the authors employed stochastic analysis of Markov chain in the closing stock price data of Oando Nigeria Plc, for the period between (2017-2021). In the same way, Amadi and Charles (2022) attempted to determine the impacts multiplicative and multiplicative inverse trends series have on the value of asset prices of investors in capital market. By adopting the separable variable and Ito's lemma respectively, the authors were able to analytically solve four investment equations. Both the graphical and computational results of stock variables and the effect on relevant parameters were elaborately discussed. In another study by Amadi et al. (2022), Examined Stochastic analysis of Markov chain. Their findings reveales that the stock (S1) has a higher mean rate of return but Stock (S2) has the best chances of price increasing in the near future hence informing the investors to have good and proper idea and of the corresponding behavior of stocks as regards decision making. Osu et al. (2022) Studied Solutions to Stochastic boundary value problems in Sobolev Spaces This paper combined two stochastic systems which gave précised conditions for determining asset values through multiplicative effects series. The validity of the analytical solution was verified using initial stock prices. Hence, this paper compliments the works of Amadi and Okpoye (2022) and Okpoye et, al., (2023) respectively by incorporating time dependent delay parameter to realistically assess asset pricing function for capital market price changes which have not been seen elsewhere. It is obvious that the previous efforts have framed a dynamic which governed asset return rates through multiplicative effects in which changes in these prices of financial asset were systematically considered. The advantage of this study over the works of Amadi and Okpoye (2022) and Okpoye et, al., (2023) is that the present work models the impact of Stochastic Delay Differential Equation (SDDE) over Stochastic

Differential Equation (SDE) through asset value changes. To the best of the researcher's knowledge this novel contribution compliments the previous discovery and extends the frontier of knowledge in this area of mathematical Finance.
The paper is set as follows: Section 2.1 is Materials and method; Section 3.1 presents results and discussion while the paper is concluded in Section 4.1.

2.1 Materials and Method

In the derivation of the stochastic model representing asset value with or without delay, some theorems and definitions are considered. They are as follows:

Definition 1: Stochastic process: A stochastic process $X(t)$ is a relation of random variables $\{X_t(\gamma), t\in T, \gamma\in\Omega\}$, that is, for each t in the index set T, $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 2: Random Walk: There are different methods to which we can state a stochastic process and then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from point $x=a$ to point $x=b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0=0$, defined by

$$S_n=\sum_{k=1}^{n} X_k \tag{1.1}$$

where X_k are independent and identically distributed random variables.

Definition 3 : A Stochastic Differential Equation is a differential equation with stochastic term. Therefore, assume that $(\Omega, F, \wp)$ is a probability space with filtration $\{f_t\}_t \geq 0$ and $W(t)=(W_1, W_1, \ldots, W_m)^T, t\geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g as follows:

$$dX = f(t,X)\,dt + g(t,X)\,dZ, \quad 0\leq t\leq T \tag{1.2}$$

$$X(0)=x_0 \tag{1.3}$$

where $T>0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f:[0,T]\times R^n$ and $g:[0,T]\times R^n \to R^{m\times n}$. SDE can also be written in the form of integral as follows:

$$X(t)=x_0+\int_0^t f(S,X(S))\,dS+\int_0^t g(S,X(S))\,dZ(S) \tag{1.4}$$

where dX and dZ are known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$

Theorem 1.1: (Ito's lemma): Let $f(S,t)$ be a twice continuous differential function on

$[0,\infty) \times A$ and let S_t denotes an Ito's process

$dS_t = a_t dt + b_t dz(t),\ t > 0$

Applying Taylor series expansion of F gives:

$F_t = F(S_t, t)$

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}(dS_t)^2 + ..$$

Ignoring the higher order in equation we have:

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + b_t dz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + b_t dz(t)\right)^2$$

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + b_t dz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 dt$$

$$dF_t = \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2\right) dt + \frac{\partial F}{\partial S_t} b_t dz(t)$$

More so, given the variable $S(t)$ denotes stock price, then following GBM implies and hence, the function $F(S,t)$, then the Ito's lemma gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S \frac{\partial F}{\partial S} dz(t)$$

2.2.1 Formulation of the Problem

We consider stochastic differential equations imposed with the dynamics of return rates which is said to have a complete probability space $(\Omega, F, \wp)$ such that a finite time investment horizon $T > 0$. Time delay is paramount in assessing asset value so adding delay term in a model gives more realistic and better model. Therefore, delay term is added to the non-Brownian term of the stochastic differential equation, stated below:

$$dX_i = (t - \tau) X_i(t) dt + \sigma X_i(t) dZ \tag{1.5}$$

where μ is an expected rate of returns on stock, σ is the volatility of the stock , dt is the relative change in the price during the period of time and dZ is a Wiener process. Following Amadi and Okpoye (2022), Amadi and Charles (2022), the rate of returns gives as:

$$\left.\begin{aligned} R_t &= (\theta_1\theta_2)^2 \\ R_t &= (\theta_1 + \theta_2)^2 \end{aligned}\right\} \tag{1.6}$$

Using equation (1.6) in equation (1.5), we obtained the following system of delay stochastic differential equations:

$$dX_1 = (t - \tau)(\theta_1\theta_2)^2 X_1(t) dt + \sigma X_1(t) dZ^1(t) \tag{1.7}$$

$$dX_3 = (\theta_1\theta_2)^2 X_3(t) dt + \sigma X_3(t) dZ^3(t) \tag{1.8}$$

The initial conditions are:

$$X_1(0) = X_0, X_3(0) = X_0 \tag{1.9}$$

where, $X_1(t)$ *and* $X_3(t)$ are asset prices, The expression dZ, which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion, λ_1 and λ_2 represents rate of returns of first and second investments respectively, $\lambda_1\lambda_2$ is the multiplicative effects.

2.2.2 Method of Solution

The model equations (1.7) and (1.8) are stochastic systems whose solutions are not trivial. We implement the methods of Ito's lemma in solving for $X_1(t), X_2(t), X_3(t),$ and $X_4(t)$. To solve this problem, we note that we can forecast the future worth of the asset with sureness.

From equation (1.7) let $f(X_1, t) = \ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_1} = \frac{1}{X_1} \tag{1.10}$$

$$\frac{\partial^2 f}{\partial X_1^2} = -\frac{1}{X_1^2} \tag{1.11}$$

$$\frac{\partial f}{\partial t} = 0 \tag{1.12}$$

Thus; from Ito's lemma, we have

$$df(X_1, t) = \frac{\partial f}{\partial X_1} dX_1 + \frac{1}{2}\frac{\partial^2 f}{\partial X_1^2}(dX_1)^2 + \frac{\partial f}{\partial t} dt \tag{1.13}$$

$$df(X_1, t) = \sigma X_1 \frac{\partial f}{\partial X_1} dZ^1 + \left[(t-\tau)(\theta_1\theta_2)^2 X_1 \frac{\partial f}{\partial X_1} dt\right] + \frac{1}{2}\frac{\partial^2 f}{\partial X_1^2}\left((t-\tau)(\theta_1\theta_2)^2 X_1 dt + \sigma X_1 dZ^1\right)^2 + \frac{\partial f}{\partial t} dt \tag{1.14}$$

$$df(X_1, t) = \sigma X_1 \frac{\partial f}{\partial X_1} dZ^1 + \left[(t-\tau)(\theta_1\theta_2)^2 X_1 \frac{\partial f}{\partial X_1} dt\right]$$

$$+\left[(t-\tau)(\theta_1\theta_2)^2 X_1^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_1^2}(dt)^2 + 2\sigma(t-\tau)(\theta_1\theta_2)^2 X_1^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_1^2} dtdZ^1 + \sigma^2 X_1^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_1^2}(dZ^1)^2\right] + \frac{\partial f}{\partial t} dt \tag{1.14}$$

Since $dt \to 0, (dt)^2 \to 0,$ and $(dZ^1)^2 \to dt$, then equation (1.15) reduces to:

$$df(X_1, t) = \sigma X_1 \frac{\partial f}{\partial X_1} dZ^1 + (t-\tau)(\theta_1\theta_2)^2 X_1 \frac{\partial f}{\partial X_1} dt + \sigma^2 X_1^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_1^2} dt + \frac{\partial f}{\partial t} dt \tag{1.15}$$

Simplifying equation (1.18), we have

$$df(X_1,t) = \sigma X_1 \frac{\partial f}{\partial X_1} dZ^1 + \left((t-\tau)(\theta_1\theta_2)^2 X_1 \frac{\partial f}{\partial X_1} + \frac{1}{2}\sigma^2 X_1^2 \frac{\partial^2 f}{\partial X_1^2} + \frac{\partial f}{\partial t} \right) dt \tag{1.16}$$

Substituting equations (1.12)-(1.14) into equation (1.19), we have:

$$df(X_1,t) = \sigma \frac{X_1}{X_1} dZ^1 + \left((t-\tau)(\theta_1\theta_2)^2 \frac{X_1}{X_1} - \frac{\sigma^2 X_1^2}{2X_1^2} + \frac{\partial f}{\partial t} \right) dt \tag{1.17}$$

Simplifying equation (1.20), we have:

$$df(X_1,t) = \sigma dZ^1 + \left((t-\tau)(\theta_1\theta_2)^2 - \frac{\sigma^2}{2} \right) dt \tag{1.18}$$

We know that, $f(X_1,t) = \ln X_1$ (1.19),

Substituting equation (1.19) we have:

$$d(\ln X_1) = \sigma dZ^1 + \left((t-\tau)(\theta_1\theta_2)^2 - \frac{\sigma^2}{2} \right) dt \tag{1.20}$$

Integrating equation (3.35), we have:

$$\int_0^t d(\ln X_1) = \int \left((t-\tau)(\theta_1\theta_2)^2 - \frac{\sigma^2}{2} \right) dt + \sigma \int dZ^1 \tag{1.21}$$

$$\ln X_1 - \ln X_0 = \frac{(t-\tau)^2}{2}(\theta_1\theta_2)^2 - \frac{\sigma^2 t}{2} + \sigma Z \tag{1.22}$$

$$\ln\left(\frac{X_1}{X_0} \right) = \frac{(t-\tau)^2}{2}(\theta_1\theta_2)^2 - \frac{\sigma^2 t}{2} + \sigma Z \tag{1.23}$$

$$\frac{X_1}{X_0} = \exp\left(\frac{(t-\tau)^2}{2}(\theta_1\theta_2)^2 - \frac{\sigma^2 t}{2} + \sigma Z \right) \tag{1.24}$$

$$X_1(t) = X_0 \exp\left(\frac{(t-\tau)^2}{2}(\theta_1\theta_2)^2 - \frac{\sigma^2 t}{2} + \sigma Z \right) \tag{1.25}$$

2.2.3 Analytical Solution of Additive Effect of the rate of returns on Investment with Delay

Following equation (1.8), we let $f(X_3,t) = \ln X_3$ so that upon differentiation, we have:

$$\frac{\partial f}{\partial X_3} = \frac{1}{X_3} \tag{1.26}$$

$$\frac{\partial^2 f}{\partial X_3^2} = -\frac{1}{X_3^2} \tag{1.27}$$

$$\frac{\partial f}{\partial t} = 0 \tag{1.28}$$

Thus; from Ito's lemma, we have

$$df(X_3,t)=\frac{\partial f}{\partial X_3}dX_3+\frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}(dX_3)^2+\frac{\partial f}{\partial t}dt \tag{1.29}$$

$$df(X_3,t)=\sigma X_3\frac{\partial f}{\partial X_3}dZ+\left((t-\tau)(\theta_1+\theta_2)^2X_3\frac{\partial f}{\partial X_3}dt\right)+\frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}\left((t-\tau)(\theta_1+\theta_2)^2X_3dt+\sigma X_3dZ\right)^2+\frac{\partial f}{\partial t}dt \tag{1.30}$$

$$df(X_3,t)=\sigma X_3\frac{\partial f}{\partial X_3}dZ+\left((t-\tau)(\theta_1+\theta_2)^2X_3\frac{\partial f}{\partial X_3}dt\right)$$
$$+\left((t-\tau)(\theta_1+\theta_2)^2X_3^2\frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}(dt)^2+2\sigma(t-\tau)(\theta_1+\theta_2)^2X_3^2\frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}dtdZ+\sigma^2X_1^2\frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}(dZ)^2\right)+\frac{\partial f}{\partial t}dt \tag{1.31}$$

Since $dt\to 0,(dt)^2\to 0,\text{ and }(dZ)^2\to dt$, :

$$df(X_3,t)=\sigma X_3\frac{\partial f}{\partial X_3}dZ+(t-\tau)(\theta_1+\theta_2)^2X_3\frac{\partial f}{\partial X_3}dt+\sigma^2X_3^2\frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}dt+\frac{\partial f}{\partial t}dt \tag{1.32}$$

Simplifying

$$df(X_3,t)=\sigma X_3\frac{\partial f}{\partial X_3}dZ+\left((t-\tau)(\theta_1+\theta_2)^2X_3\frac{\partial f}{\partial X_3}+\frac{1}{2}\sigma^2X_3^2\frac{\partial^2 f}{\partial X_3^2}+\frac{\partial f}{\partial t}\right)dt \tag{1.33}$$

Substitutions we have:

$$df(X_3,t)=\sigma\frac{X_3}{X_3}dZ+\left((t-\tau)(\theta_1+\theta_2)^2\frac{X_3}{X_3}-\frac{\sigma^2X_3^2}{2X_3^2}+\frac{\partial f}{\partial t}\right)dt \tag{1.34}$$

Simplifying we have:

$$df(X_3,t)=\sigma dZ+\left((t-\tau)(\theta_1+\theta_2)^2-\frac{\sigma^2}{2}\right)dt \tag{1.35}$$

We know that, $f(X_3,t)=\ln X_3$ (1.36)

Substituting, we have:

$$d(\ln X_3)=\sigma dZ+\left((t-\tau)(\theta_1\theta_2)^2-\frac{\sigma^2}{2}\right)dt \tag{1.37}$$

Integrating , we have:

$$\int_0^t d(\ln X_3)=\int\left((t-\tau)(\theta_1\theta_2)^2-\frac{\sigma^2}{2}\right)dt+\sigma\int dZ^1 \tag{1.38}$$

$$\ln X_3-\ln X_0=\frac{(t-\tau)^2}{2}(\theta_1+\theta_2)^2-\frac{\sigma^2t}{2}+\sigma Z \tag{1.39}$$

$$\ln\left(\frac{X_3}{X_0}\right)=\frac{(t-\tau)^2}{2}(\theta_1+\theta_2)^2-\frac{\sigma^2t}{2}+\sigma Z \tag{1.40}$$

$$\frac{X_3}{X_0} = \exp\left(\frac{(t-\tau)^2}{2}(\theta_1+\theta_2)^2 - \frac{\sigma^2 t}{2} + \sigma Z\right) \tag{1.41}$$

$$X_3(t) = X_0 \exp\left(\frac{(t-\tau)^2}{2}(\theta_1+\theta_2)^2 - \frac{\sigma^2 t}{2} + \sigma Z\right) \tag{1.42}$$

3. 1 Results and Discussion

This Section presented numerical results as follows:

Table 1: The impact of time delay in the assessment of multiplicative Asset values through the solution below:

$$X_3(t) = X_0 \exp\left\{(t-\tau)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma\, dz(t)\right\} \quad t = 2, \quad dz = 1$$

τ	S_0	σ	$(\theta_1\theta_2)$	$X_3(t)$	$(\theta_1\theta_2)$	$X_3(t)$
	5.00	0.2	1.0000	188.5641	2.0000	6847799.464
0.25	6.00	0.2	1.0000	226.2769	2.0000	8217359.356
	5.70	0.2	1.0000	214.9631	2.0000	7806491.388
	5.20	0.2	1.0000	161.2021	2.0000	3212816.824
0.35	4.60	0.2	1.0000	142.6018	2.0000	2842107.19
	4.00	0.2	1.0000	124.0016	2.0000	2471397.557
	3.70	0.2	1.0000	94.2860	2.0000	1031302.186
0.45	3.90	0.2	1.0000	99.3826	2.0000	1087048.25
	3.50	0.2	1.0000	89.1895	2.0000	975556.122
	3.31	0.2	1.0000	69.3349	2.0000	416211.2361
0.55	3.75	0.2	1.0000	78.5516	2.0000	471538.4094
	4.00	0.2	1.0000	83.7884	2.0000	502974.3034

The results in Table 1 demonstrated that the various levels of asset value increase systematically as the delay parameter τ increases, under varying initial stock prices S_0 and constant volatility $\sigma = 0.2$. Specifically, small increases in return rates of 1.0000 and 2.0000 correspondingly amplify asset values over time. This finding is consistent with the intuitive reality of capital market behaviour: an investor who delays the liquidation of landed properties, company shares, or other assets allows such assets to appreciate in value before monetisation, thereby earning more than if they had sold immediately. This result aligns with the findings of Okpoye et al. (2023), who, in their empirical assessment of asset value functions for capital market price changes, demonstrated that increasing stock market prices produce a corresponding increase in asset values, and that multiplicative return structures offer the best basis for investor decision-making. Similarly, Amadi and Okpoye (2022), in their application of stochastic models to capital market investments using Dangote Cement Plc data, found that multiplicative effects serve as a key parameter in obtaining stock rate of returns a finding that the present study not only corroborates but extends by incorporating a time-dependent delay mechanism absent from their framework.

Furthermore, the observed positive relationship between return rate and asset value under the multiplicative SDDE structure is consistent with the work of Loko et al. (2023), who showed that return rates, when modelled as drift parameters in SDEs, govern price functions periodically and generate outcomes that support investor decision-making. The present study advances this by demonstrating that delay magnifies these outcomes in ways that purely SDE-based frameworks cannot capture.

Table 2: The assessment of multiplicative Asset values through the solution below: $X_1(t) = X_0 \exp\left\{\left[(\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2\right]t + \sigma dz(t)\right\}$ t = 2, $dz = 1$

S_0	σ	$(\theta_1\theta_2)$	$X_1(t)$	$(\theta_1\theta_2)$	$X_1(t)$
5.00	0.2	1.0000	43.3557	2.0000	17490.9330
6.00	0.2	1.0000	52.0268	2.0000	20989.1196
5.70	0.2	1.0000	49.4255	2.0000	19939.6636
5.20	0.2	1.0000	45.0899	2.0000	18190.5703
4.60	0.2	1.0000	39.8872	2.0000	16091.6584
4.00	0.2	1.0000	34.6846	2.0000	13992.7464
3.70	0.2	1.0000	32.0832	2.0000	12943.2904
3.90	0.2	1.0000	33.8174	2.0000	13642.9278
3.50	0.2	1.0000	30.3490	2.0000	12243.6531
3.31	0.2	1.0000	28.7015	2.0000	11578.9977
3.75	0.2	1.0000	32.5168	2.0000	13118.1998
4.00	0.2	1.0000	34.8846	2.0000	13992.7464

Table 2 showed that the initial stock price S_0, when combined with fixed return rates of 1.0000 and 2.0000, determines the high levels of asset value that an investor can accumulate over time. The SDDE model consistently produced higher asset valuations than the corresponding SDE model, demonstrating that the memory-dependent structure of the delay differential equation captures additional wealth accumulation dynamics. This finding supports and extends the work of Amadi and Charles (2022), who obtained precise conditions controlling asset price rates through multiplicative and multiplicative inverse trend series using Itô's lemma, and confirmed their results both graphically and computationally. While Amadi and Charles (2022) restricted their analysis to SDE frameworks, the present study demonstrates that when a delay parameter is superimposed onto this structure, the resulting SDDE consistently outperforms the standard SDE in tracking realistic asset value trajectories. The result is also in agreement with Adeosun et al. (2015), who applied stochastic analysis to stock market price models using a log-normal distribution model formulated as an SDE and confirmed its efficiency in forecasting stock prices. The SDDE framework employed in the present study offers a richer extension of this approach, explicitly accommodating the time-lag effects that Adeosun et al. (2015) did not model.
Moreover, the finding that higher initial stock prices amplify asset value growth over time corroborates Agwuegbo et al. (2016), who demonstrated that the Nigerian Stock Market index follows a geometric Brownian motion process, confirming that initial price conditions significantly shape long-run asset trajectories. The present study reinforces this conclusion within the Oando Plc context, adding that delay further magnifies this initial price sensitivity.

4.2. Goodness of Fit test

H_0: The asset value functions $X_1(t)$ and $X_3(t)$ comes from a common distribution

H_1: They are not from a common distribution

Table 3: Goodness of fit test for assessment of two asset values with and without delay when its return rates is: 1.0000

$(\theta_1\theta_2)$	$X_3(t)$	$X_1(t)$	Mean	STD	KS Stat	P-value	Decision
1.0000	43.3557	188.5641	115.9599	102.6778	1.0000	0.2890	Accept
1.0000	52.0268	226.2769	139.1518	123.2134	1.0000	0.2890	Accept
1.0000	49.4255	214.9631	132.1943	1117.0528	1.0000	0.2890	Accept
1.0000	45.0899	161.2021	103.1460	82.1037	1.0000	0.2890	Accept
1.0000	39.8872	142.6018	91.2445	72.6302	1.0000	0.2890	Accept
1.0000	34.6846	124.0016	79.3431	63.1567	1.0000	0.2890	Accept
1.0000	32.0832	94.2860	63.1846	43.9840	1.0000	0.2890	Accept
1.0000	33.8174	99.3826	66.6000	46.3616	1.0000	0.2890	Accept
1.0000	30.3490	89.1895	59.7692	41.6065	1.0000	0.2890	Accept
1.0000	28.7015	69.3349	49.0182	28.7322	1.0000	0.2890	Accept
1.0000	32.5168	78.5516	55.5342	32.5515	1.0000	0.2890	Accept
1.0000	34.8846	83.7884	59.3365	34.5802	1.0000	0.2890	Accept

The Kolmogorov–Smirnov (KS) goodness-of-fit tests reported in Tables 3 and 4 accepted the alternative hypothesis at $\alpha = 0.01$, confirming that asset values with delay and without delay do not originate from the same statistical distribution. This finding has significant financial implications: it establishes that delay is not merely a quantitative enhancement but a structurally transformative mechanism that fundamentally alters the stochastic behaviour of asset values.

The graphical representations in Figures 4.1 and 4.2 corroborate this finding. Figure 4.1 shows that at a return rate of 1.0000, the delayed asset value trajectory rises sharply — indexed in millions of Naira — while the non-delayed trajectory remains low and stagnant. Figure 4.2 confirms that at a return rate of 2.0000, the delayed plot maintains an upward trend despite inherent stochastic fluctuations, while the non-delayed plot hovers near the origin along the horizontal axis.These findings are consistent with the conclusions of Liu et al. (2013), who modelled delayed reactions in financial markets and found that time-lag structures significantly alter the dynamics of asset pricing. The present study provides empirical support for this position, demonstrating with real Oando Plc data that delay systematically shifts asset value distributions upward. Equally, the results agree with Zhou et al. (2016), who studied market reaction delay and asset price dynamics and found that delayed market responses produce structurally distinct price trajectories — a finding directly mirrored in the KS test results of the present study.

The mean and standard deviation values in column 5 of Tables 3 and 4 further reveal that higher return rates (2.0000) amplify the already superior performance of delayed asset values relative to non-delayed ones. This interplay between delay and return rate echoes the findings of Wenli et al. (2014), who demonstrated that when a stochastic differential equation is exponentially stable and time delay is sufficiently small, the mixed-delay SDE retains stability — confirming that delay, when properly modelled, enhances rather than destabilises financial outcomes.

Table 4: Goodness of fit test for assessment of two asset values with and without delay when its return rates is: 2.0000

$(\theta_1\theta_2)$	$X_1(t)$	$X_1'(t)$	Mean	STD	KSStat	P-value	Decision
2.0000	17490.9330	6847799.464	3.1326e+06	4.8298e+06	1.0000	0.2890	Accept
2.0000	20989.1196	8217359.356	4.1192e+06	5.7957e+06	1.0000	0.2890	Accept
2.0000	19939.6636	7806491.388	3.9132e+06	5.5059e+06	1.0000	0.2890	Accept
2.0000	18190.5703	3212816.824	1.6155e+06	2,2589e+06	1.0000	0.2890	Accept
2.0000	16091.6584	2842107.19	1.4291e+06	1.9983e+06	1.0000	0.2890	Accept
2.0000	13992.7464	2471397.557	1.2427e+06	1.7376e+06	1.0000	0.2890	Accept
2.0000	12943.2904	1031302.186	5.2212e+05	7.2009e+05	1.0000	0.2890	Accept
2.0000	13642.9278	1087048.25	5.5035e+05	7.5901e+05	1.0000	0.2890	Accept
2.0000	12243.6531	975556.122	4.9390e+05	6.8116e+05	1.0000	0.2890	Accept
2.0000	11578.9977	416211.2361	2.1390e+05	2.8612e+05	1.0000	0.2890	Accept
2.0000	13118.1998	471538.4094	2.4233e+05	3.2415e+05	1.0000	0.2890	Accept
2.0000	13992.7464	502974.3034	2.5848e+05	3.4576e+05	1.0000	0.2890	Accept

The study considered the Goodness of fit test for assessment of two asset values with and without delay when its return rates is 2.000. this indicates that there is significant difference between the two asset values $(X_1(t), X_3(t))$ in time varying investments.

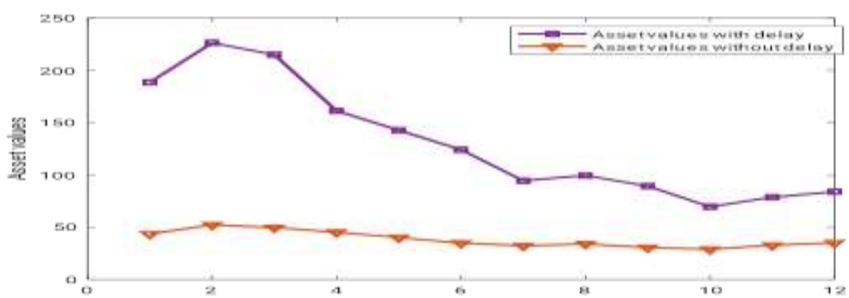

Figure 1: An assessment of asset value function when return rates is 1.0000. This describes the financial assessment of asset values with delay and without delay respectively when return rates is 1.0000. The two plots are independently separated and also not correlated

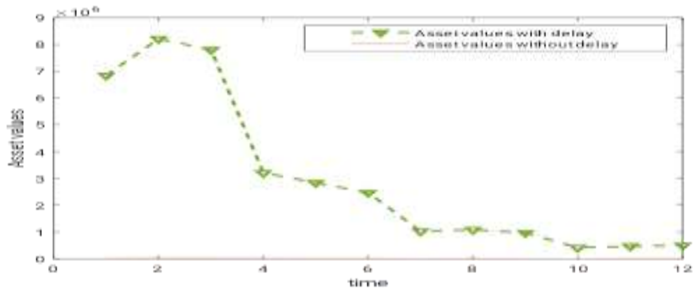

Figure 2: An assessment of asset value function when return rates is 2.0000
In Figure 2, it can be seen that the two-asset value function are quite not related in terms of flow in capital investments. The delay plot shows upward trend despite the of its movements.

4.1 Conclusion
This study has demonstrated the crucial role of stochastic modeling in estimating and analyzing asset value changes within dynamic and volatile capital markets, such as that of Oando PLC. Through the application of both Stochastic Differential Equations (SDE) and Stochastic Delay Differential Equations (SDDE), the research revealed that incorporating delay parameters significantly enhances the accuracy of asset valuation. The findings indicate that: An increase in the delay parameter dominantly increases asset value due to the cumulative effects of past price movements. The SDDE model offers more realistic asset value estimations than the standard SDE because it accounts for memory effects and market inertia. Higher initial stock prices amplify the trajectory of asset value growth over time.
Variability in asset values (as reflected by standard deviation) directly corresponds to volatility levels, implying that high-value assets exhibit larger fluctuations.
The Kolmogorov-Smirnov (KS) test confirmed that asset values, with and without delay, do not originate from a common distribution, underscoring the distinct dynamics captured by the delay-inclusive model. Overall, this research underscores that the inclusion of stochastic delay structures provides a more comprehensive understanding of real-world market behavior, offering improved predictive capability for investors and analysts in the oil and gas sector.

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